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06MAT41

Fourth Semester B.E. Degree Examination, December 2012
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Given that $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$, to find an approximate value of y at x = 0.1 and x = 0.2 by Taylor's series method. **(06 Marks)**

b. Using Euler's modified method, solve for y at x = 0.1 if $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1, carryout three modifications. **(07 Marks)**

c. Given $\frac{dy}{dx} = (1+y)x^2$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979, determine y(1.4) by Adams – Bash forth method. **(07 Marks)**

2 a. Show that an analytic function with constant modulus is constant. **(06 Marks)**

b. Find the analytic function $f(2) = u + iv$, if $u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$ **(07 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, -1 into the points w = i, 0, -i and hence find the image $|z| < 1$. **(07 Marks)**

3 a. Using the Cauchy's integral formula, to evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where c : $|z| = 3$. **(06 Marks)**

b. Obtain the Laurent's series for the function $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ in the regions i) $2 < |z| < 3$ ii) $|z| > 3$. **(07 Marks)**

c. Determine the poles of $\frac{z^2}{(z-1)^2(z+2)}$ and the residues at each pole. **(07 Marks)**

4 a. Prove that $e^{x/2(t - 1/t)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. **(06 Marks)**

b. Show that $J_n(x) = \frac{x}{2n}\{J_{n+1}(x) + J_{n-1}(x)\}$ **(07 Marks)**

(07 Marks)

c. Explain the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre's polynomials.

PART – B

5 a. Fit a straight line to the following data: **(06 Marks)**

x:	0	1	2	3	4
y:	1.0	1.8	3.3	4.5	6.3

5 b. Prove that $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$, where γ, σ_x, σ_y have their usual meanings and explain the significance of $r = \pm 1$ and $r = 0$. **(07 Marks)**

c. A certain problem is given to four students for solving. The probability of their solving the problem are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(07 Marks)**

6 a. The probability density function P(x) of a continuous random variables is given by, $P(x) = y_0 e^{-|x|}$, $-\infty < x < \infty$, prove that $y_0 = \frac{1}{2}$. Find the mean and variance of the distribution. **(06 Marks)**

b. Derive the mean and variance of the binomial distribution. **(07 Marks)**

c. If x is an exponential variate with mean 4, evaluate i) $P(0<x<1)$ ii) $P(x>2)$ and iii) $P(-\infty<x<10)$. **(07 Marks)**

7 a. Define the terms: i) Null hypothesis ii) Level of significance and iii) Confidence limits. **(06 Marks)**

b. A sugar factory is expected to sell sugar in 100 kg bags. A sample of 144 bags taken from a day's output shows the average and S.D. of weights of these bags as 99 and 4 kg respectively. Can we conclude that the factory is working as per standards? (Table value of $z = 1.96$ at 5% Log) **(07 Marks)**

c. The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accident are uniformly distributed over the week. ($X^2_{0.05} = 9.41$ for 4 d.f.) **(07 Marks)**

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accident	14	16	8	12	11	9	14	84

8 a. The joint probability distribution for the following table:

x \ y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of x and y and verify that x and y are independent variables. **(06 Marks)**

b. Find the fixed probability vector of the following regular stochastic matrix.

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}.$$ **(07 Marks)**

c. Define the following terms:
i) Regular state ii) Periodic state iii) Recurrent state and iv) Transient state. **(07 Marks)**

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